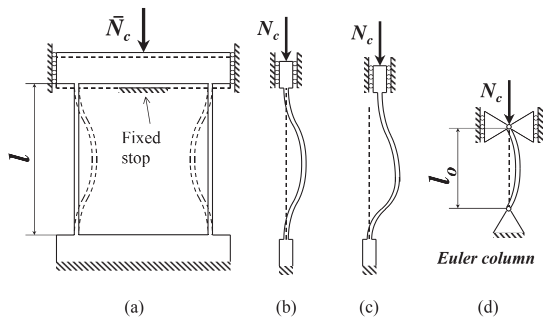


**Figure 4.4** Buckling of the two parallel leaf spring stage. When the compressive load exerted on the table reaches the critical load, the leaf springs buckle, taking the shape illustrated in (a). The leaf springs may buckle inward or outward at random. When the leaf springs buckle, the system becomes unstable. To be able to observe it in this state, a fixed stop must be added. The critical load is independent of the deflection $f$ of the leaf spring: the critical load of leaf springs (b) and (c) is the same. The Euler column equivalent to load case (b) is represented by (d).

It is vital to remember here that the buckling model that we use makes the assumption that the beam remains within the elastic deformation range for all loads ranging up to the critical load $0 \leqslant \bar{N} < \bar{N}_c$. Thus, the critical load that we have just calculated forms a threshold beyond which the system is inevitably destroyed by a phenomenon of elastic instability regardless of the elastic limit of the material used. In actual fact, the material used has a finite elastic limit. As a result, depending on the case, the system could be destroyed by the onset of stresses in the leaf springs that exceed the stress allowed by them, for loads below the critical load, i.e., before elastic instability occurs. At this stage of our development, we can draw just one conclusion – the allowable load of a table with parallel leaf springs is always less than its critical load:

$$\bar{N}_{adm} \leqslant \frac{8\pi^2 EI}{l^2}. \tag{4.20}$$

At the end of this section we will prove that for small deflections in relation to the maximum deflection allowed by the two parallel leaf spring stage, the allowable load is well and truly equal to the critical load. However, for larger deflections (close to $f_{adm}$), the elastic limit is exceeded at loads below $\bar{N}_c$.

### Tensile/compressive behaviour in the elastically stable domain

Let us consider once again that the force $\bar{P}$ is applied to point B and that it thus does not induce any tensile/compressive forces in the leaf springs: $T_1 = T_2 = 0$. This force causes the mobile block to move through a distance $f$, which this

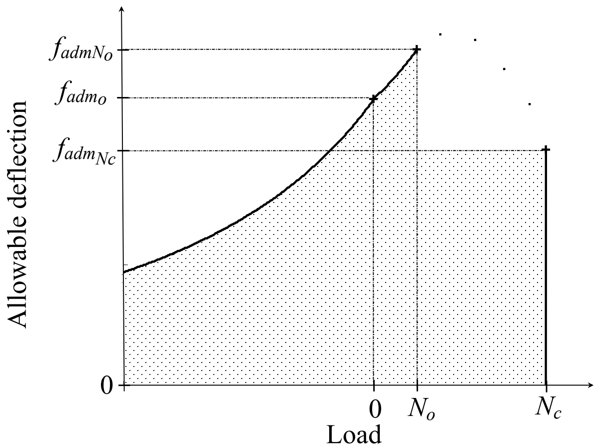


**Figure 4.11** Allowable deflection as a function of load. This function comprises four sections. The first, for $N < 0$, is defined by (4.51). The second, for $0 \leqslant N < N_o$, is defined by (4.52). The third, for $N_o < N < N_c$, has not been calculated. All that we know about it is that it is located above $f_{adm}(N_c)$. The fourth section, for $N = N_c$, is a vertical straight line defined by (4.58). This figure corresponds to a case for which condition (4.55) is fulfilled. This graph may also be construed as a plot of the allowable load as a function of deflection. We have shown that all the load and deflection combinations corresponding to the shaded area do not produce stresses greater than the allowable stress.

From (4.33) and (4.39) we know that

$$P = Kf = (K_o - \frac{K_o}{N_o}N)f,$$

which allows us to eliminate $P$ from equation (4.45) and find the moment at the beam's clamped ends as a function of the deflection $f$ and the load $N$:

$$M = f\left(\frac{N}{2} + \frac{K_o l}{2} - \frac{K_o N l}{2N_o}\right). \tag{4.46}$$

According to (2.3), this moment, in applying a bending load on the leaf spring, induces a stress

$$\sigma_{flex} = \frac{6M}{bh^2}. \tag{4.47}$$

The force $N$ also applies a compressive load on the leaf spring and induces a stress

$$\sigma_{comp} = \frac{N}{bh}. \tag{4.48}$$

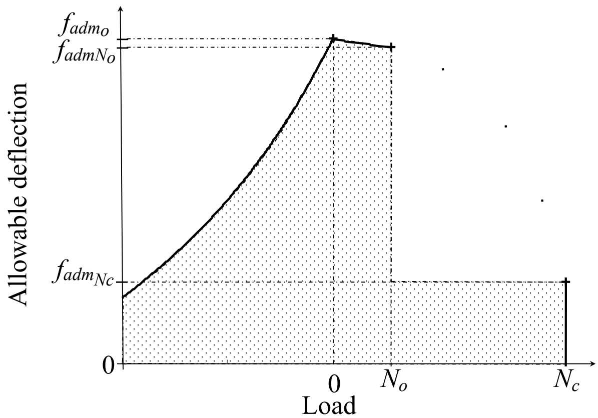


**Figure 4.12** Like Fig. 4.11, this figure presents the allowable deflection as a function of load. But, unlike the case in the previous figure, the condition (4.55) has here not been fulfilled. The consequence is that the allowable deflection decreases as the load increases between 0 and $N_o$.

The maximum stress that occurs in the leaf spring is the sum[11] of the stresses due to the bending moment and the compressive force

$$\sigma = \sigma_{flex} + |\sigma_{comp}|. \tag{4.49}$$

By combining (4.46), (4.47), (4.48) and (4.49), we obtain the stress as a function of load and deflection:

$$\sigma = 3f\left(\frac{Eh}{l^2} + \frac{N(\pi^2 - 12)}{bh^2\pi^2}\right) + \left|\frac{N}{bh}\right|. \tag{4.50}$$

By isolating $f$ in this equation, we can deduce the formula for the allowable deflection as a function of the allowable stress (Fig. 4.11 and Fig. 4.12):

$$\boxed{f_{adm} = \frac{hl^2\pi^2(N + bh\sigma_{adm})}{3bEh^3\pi^2 + 3l^2N(\pi^2 - 12)} \quad \text{valid for } N < 0} \tag{4.51}$$

$$\boxed{f_{adm} = \frac{hl^2\pi^2(-N + bh\sigma_{adm})}{3bEh^3\pi^2 + 3l^2N(\pi^2 - 12)} \qquad \text{valid for} \qquad 0 \leqslant N < N_o}$$

(4.52)

[11] The term $\sigma_{flex}$ is always positive regardless of the sign of $N$. As for the stress $\sigma_{comp}$, it is negative when the load exerts a tensile stress on the leaf springs (negative $N$). This is why the absolute value is used in (4.49).